# Chapter 1

# Experimental Foundations

### Lecture 1: A first glimpse of superposition

2026.09.02

Quantum mechanics is motivated by reproducible experiments that cannot be described consistently using classical particles or classical waves. Two familiar observations already show that classical physics is incomplete:

> **Questions.**
> - Why does a classically orbiting electron not radiate away its energy and collapse into the atomic nucleus?
> - Why can even very intense light fail to eject electrons from a metal below a certain threshold frequency?

This lecture introduces idealized binary measurements and uses a path-recombination experiment to motivate superposition. The discussion is entirely experimental; state vectors and wave functions will be introduced later.

## 1.1 Binary measurement boxes

**Idealized apparatus** A binary measurement box has one input port and two output ports. A quantum system enters the box, one of two possible outcomes is obtained, and the system leaves through the corresponding output port. The box describes the observable input-output behavior of a measurement, not the internal engineering of a particular apparatus.

**Outcome labels** We consider two binary measurements called *color* and *hardness*. The color outcomes are Black and White, while the hardness outcomes are Hard and Soft. These names are arbitrary labels and do not refer to optical color or mechanical hardness.

**Interpreting the diagrams** "In" denotes the input port. Electrons are sent through the apparatus one at a time. A percentage is found by repeating the same preparation and measurement many times and counting how often each output occurs; **it does not describe one electron splitting into fractions**.

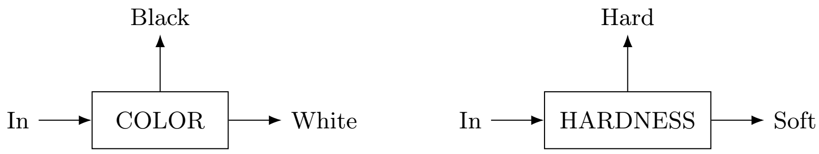


Figure 1.1: Color and hardness boxes.

**Physical realization** A real apparatus can be more complicated. For example, a Stern–Gerlach apparatus uses an inhomogeneous magnetic field to separate a beam of spin-carrying particles into two output beams. Differently oriented apparatuses can realize the two measurements represented here by color and hardness. We will use the boxes to focus on outcomes and probabilities rather than experimental engineering.

## 1.2 Basic properties of the measurement boxes

We first establish two experimental facts about the boxes: an immediate repetition of the same measurement preserves its outcome, while the outcome of one measurement does not predict the outcome of the other.

### 1.2.1 Repeatability

**Procedure** Place two identical boxes in sequence. Select one output of the first box and send only the electrons from that output into the second box of the same type.

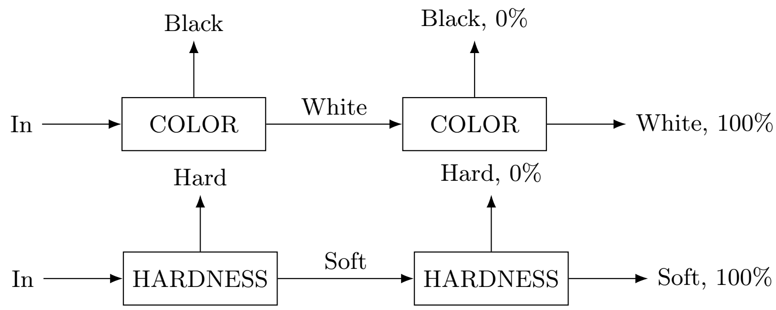


Figure 1.2: A color or hardness measurement followed by the same measurement is repeatable.

**Definition 1** (Repeatability)**.** A measurement is *repeatable* if an immediate repetition gives the same outcome with probability one.

**Observation** Selecting White and immediately measuring color again gives White with probability one. The same result holds for Black, Hard, and Soft. Thus an output of a measurement box prepares a definite result for an immediate repetition of that measurement.

### 1.2.2 Uncorrelated color and hardness measurements

**Procedure** Select the Soft output of a hardness box and send those electrons into a color box.

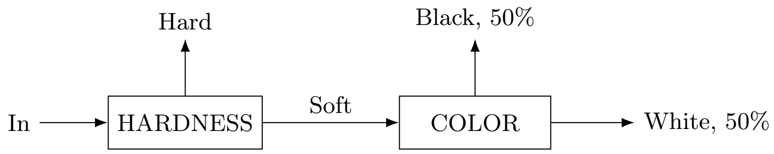


Figure 1.3: A definite hardness result does not determine the result of a color measurement.

**Observation** The selected Soft electrons split evenly at the color box:

$$P(\text{Black} \mid \text{Soft}) = P(\text{White} \mid \text{Soft}) = \frac{1}{2}. \tag{1.1}$$

The same result holds if Hard is selected instead. Reversing the order gives the corresponding result: electrons selected from either color output split evenly when hardness is measured. The following figure shows the case in which Black is selected.

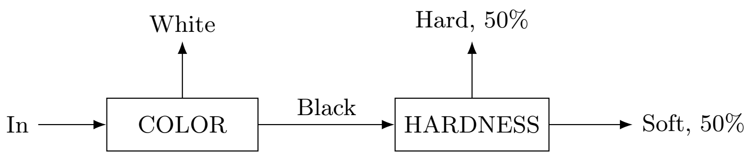


Figure 1.4: Reversing the order gives the same 50–50 distribution.

**Definition 2** (Uncorrelated measurements)**.** Two measurements are *uncorrelated* if selecting either outcome of one measurement leaves the two outcomes of the other measurement equally probable.

The reproducible 50–50 distributions therefore show that color and hardness are uncorrelated: a definite result for one does not predict the result of the other.

## 1.3 Sequential measurements

We next place color and hardness boxes in sequence to determine whether a measurement of one property affects a later measurement of the other.

### 1.3.1 An intermediate measurement changes the result

**Procedure** Select White electrons with the first color box, send them through a hardness box, select the Soft output, and then measure color again.

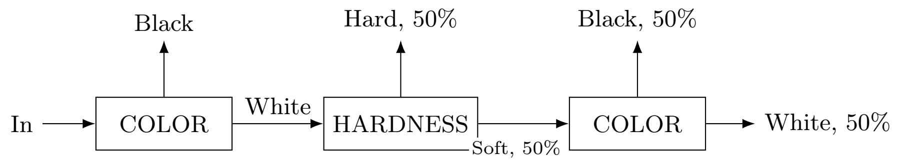


Figure 1.5: An intermediate hardness measurement changes the result of the final color measurement.

**Observation** The final color measurement gives 50% Black and 50% White, although the first color result was White. Selecting Hard instead of Soft, or starting with Black instead of White, gives the same 50–50 final distribution. If the roles of color and hardness are reversed, an intermediate color measurement likewise changes a definite hardness result into a 50–50 distribution.

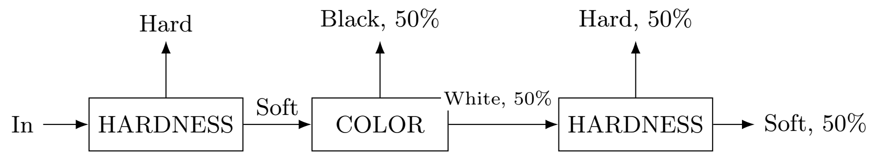


Figure 1.6: An intermediate color measurement changes the result of the final hardness measurement.

**Conclusion** Repeatability applies only when the same measurement is repeated without an intervening measurement of the other property. Selecting a hardness outcome removes the earlier definite color result, and selecting a color outcome removes the earlier definite hardness result. Individual final outcomes vary, but the 50–50 distribution is reproducible.

### 1.3.2 Failure of a joint color–hardness box

**Proposed construction** A joint color–hardness box would have four outputs, labeled White–Hard, White–Soft, Black–Hard, and Black–Soft. Each output would need to pass both repeatability tests: a subsequent color measurement would reproduce its color label, and a subsequent hardness measurement would reproduce its hardness label. A natural attempt is to place a color box first and a hardness box on each color output.

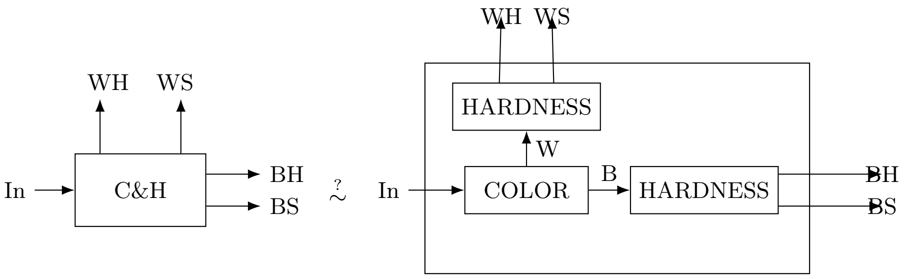


Figure 1.7: A proposed C&H box (left) and its attempted construction from one color box and two hardness boxes (right).

**Test** Select the output labeled Black–Hard. An immediate hardness measurement returns Hard with probability one, but a color measurement gives 50% Black and 50% White. The output therefore prepares a definite hardness, not simultaneous definite values of color and hardness. Reversing the order of the two boxes makes color repeatable but removes definite hardness.

**Conclusion**

$$\boxed{\text{A definite hardness state is not a definite color state.}}$$

Equivalently,

$$\boxed{\text{Color and hardness should not be treated as simultaneous classical labels.}}$$

## 1.4 Superposition and two possible paths

We now compare a completed hardness measurement with a reversible apparatus that separates the Hard and Soft paths and then recombines them. The difference between these procedures provides the first evidence for superposition.

### 1.4.1 Separating and recombining the two possible paths

**Apparatus** A hardness box separates the electron beam into a Hard path and a Soft path. Two mirrors redirect the paths without changing color or hardness, and a combining mirror returns them to one output path. The entire region inside the large rectangle is treated as one apparatus: neither hardness output is selected or discarded before the paths are recombined.

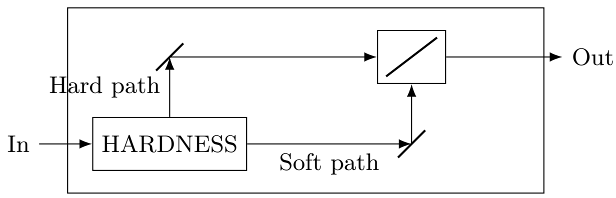


Figure 1.8: Inside the apparatus, two mirrors redirect the Hard and Soft paths, and a combining mirror brings them back into one output path.

Three experiments establish the behavior of this apparatus.

**Experiment 1: Measure hardness** Send in White electrons and measure hardness after recombination. The result is 50% Hard and 50% Soft.

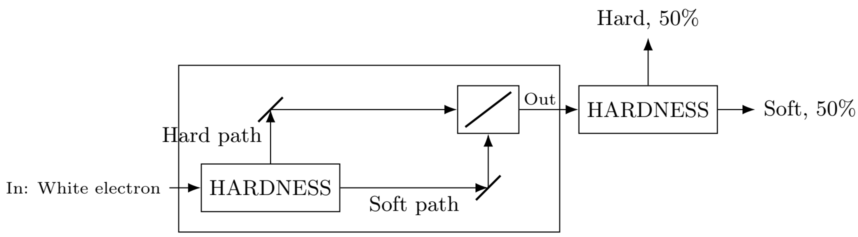


Figure 1.9: Experiment 1: White incident electrons are recombined and then measured for hardness.

**Experiment 2: Measure the color of a Hard input** Send in Hard electrons and measure color after recombination. The result is 50% Black and 50% White.

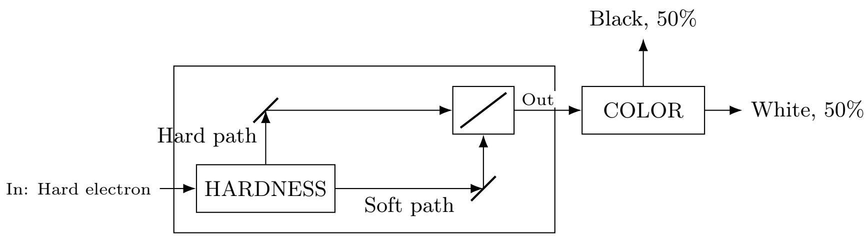


Figure 1.10: Experiment 2: Hard incident electrons are recombined and then measured for color.

**Experiment 3: Recombine a White input** Send in White electrons, leave both paths open, and measure color after recombination. The result is 0% Black and 100% White.

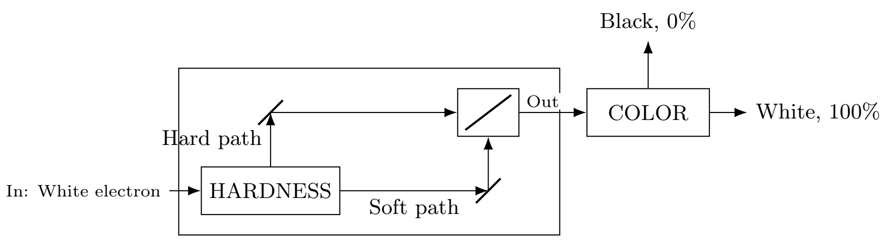


Figure 1.11: Experiment 3: White incident electrons are recombined and then measured for color.

Experiment 3 is the key observation. Either path by itself leads to a 50–50 color distribution, but the two open paths together return the initial White result with probability one.

**Completed measurement versus reversible separation** For comparison, recall the earlier experiment in which the Soft output of a hardness measurement was selected between two color measurements.

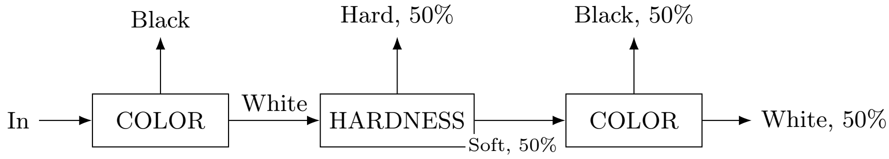


Figure 1.12: For comparison, selecting the Soft output of an intermediate hardness measurement gives a 50–50 final color distribution.

The two procedures differ at the hardness stage:

- In the earlier experiment, only the Soft output is sent to the final color box; the Hard output is discarded. The selected electrons are therefore prepared as Soft, and a color measurement gives 50% Black and 50% White.
- In Experiment 3, neither path is selected or discarded. The labels Hard and Soft identify two paths inside the larger apparatus, and both paths are brought together before the final color measurement. The result is 0% Black and 100% White.

The combining mirror therefore does not restore a color value after a completed hardness measurement. In Experiment 3, no hardness output is selected or discarded. The apparatus keeps both paths available and reverses their spatial separation before color is measured.

### 1.4.2 Blocking one path

**Procedure** Repeat Experiment 3 with an absorbing barrier in the Soft path. Send in White electrons and measure color at the output.

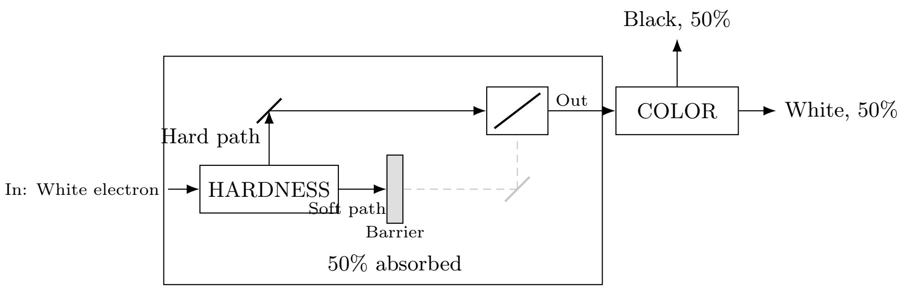


Figure 1.13: An absorbing barrier blocks the Soft path. The 50–50 color percentages are conditional on the electrons that reach the final color box.

**Observation** The percentages can be reported in two equivalent ways:

- The barrier absorbs 50% of the incident electrons.

- The remaining 50% reach the final color box. Among these surviving electrons, 50% are Black and 50% are White.
- Relative to all incident electrons, the outcomes are therefore 50% absorbed, 25% Black, and 25% White.

**Conclusion** With both paths open, the final result is White with certainty. With the Soft path blocked, only the Hard path reaches the final color box, so the transmitted electrons give a 50–50 color distribution. Blocking the Hard path gives the same conclusion with the roles of the paths reversed.

### 1.4.3 Comparison with an ordinary statistical mixture

A classical statistical mixture would say that each electron takes exactly one path, with probabilities $P(H) = P(S) = 1/2$, and that we simply do not know which path it takes. The blocking experiments show that either path by itself gives $P(B \mid H) = P(B \mid S) = 1/2$. Ordinary conditional probability would therefore predict

$$P(B) = P(B \mid H)P(H) + P(B \mid S)P(S) = \frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{2} = \frac{1}{2}. \tag{1.2}$$

Experiment 3 instead gives $P(B) = 0$ and $P(W) = 1$. The two open paths therefore cannot be represented as an ordinary random choice between a Hard-path electron and a Soft-path electron. Both paths must be retained together in the quantum description.

**Definition 3** (Superposition: preliminary form)**.** When the two possible paths must be kept together and their combined result differs from a classical statistical mixture, we call the state a *superposition* of the two paths. In this experiment, a White electron between the hardness box and the combining mirror is in a superposition of the Hard and Soft paths.

A superposition does not mean that the electron breaks into two pieces. It means that both possible paths contribute to the final result. Later, probability amplitudes will describe how these contributions combine to give White with probability one.

The same type of behavior has been observed with photons, neutrons, atoms, and large molecules. Superposition is therefore a general feature of quantum systems, not a special property of electrons.

**Questions.**

Why do two open paths give 100% White, while either path alone gives a 50–50 color distribution?

For a single electron with both paths open, which path did it take?

Hard path? No. Soft path? No.
Both paths? No. Neither path? No.

*Suggested reading.*

- Feynman, Leighton, and Sands, *The Feynman Lectures on Physics*, Vol. III, Ch. 1, *Quantum Behavior*, Secs. 1–4 through 1–7. Optional background: Secs. 1–2 and 1–3.
- Susskind and Friedman, *Quantum Mechanics: The Theoretical Minimum*, Ch. 1, *Systems and Experiments*, Secs. 1.1–1.4, especially Sec. 1.3, *An Experiment*, and Sec. 1.4, *Experiments Are Never Gentle*.